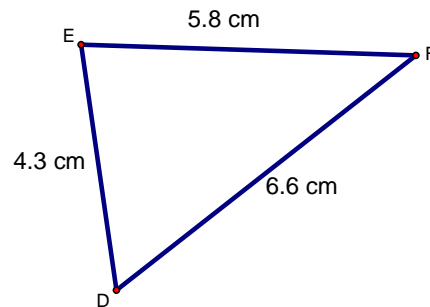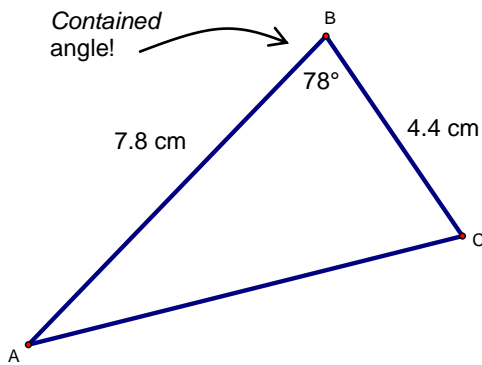# The Cosine Law

Part A - Concept

When you are given an acute, non-right angle triangle, you can use the sine law as long as you know either two sides and an angle across from a known side, or two angles and any side.

Consider acute, non-right angled $\triangle ABC$ and $\triangle DEF$:

In $\triangle ABC$, you are given two side lengths and a *contained* angle. The problem is that the given sides are not across from the given angle. Therefore, the sine law cannot be used.

In $\triangle DEF$, you are given three side lengths but no angles. In order to use the sine law, there must be at least one angle that is known. Therefore, the sine law cannot be used.

We need a new method!

Given acute $\triangle ABC$,
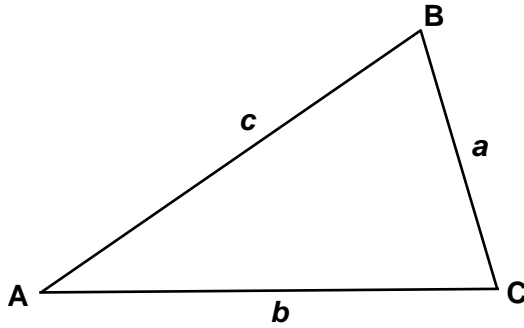
THE COSINE LAW can be used to solve a triangle if you know either:

      1) two sides and a *contained* angle, OR

      2) three sides

The Cosine Law:

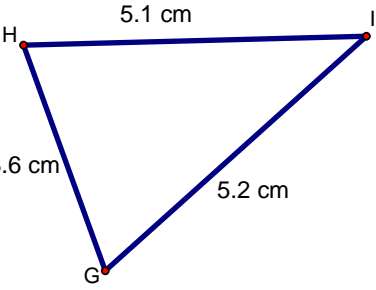To solve for the third side given two sides and a *contained* angle:
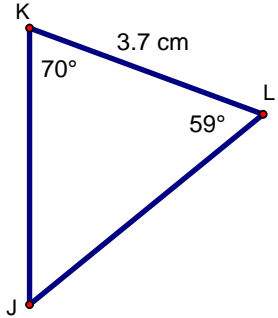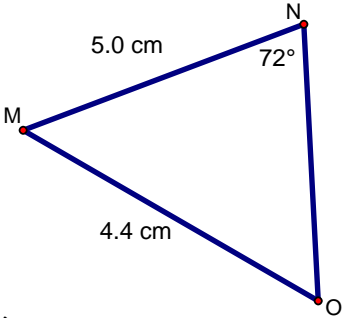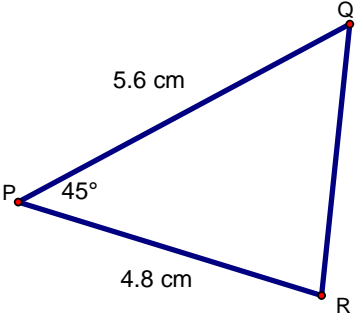
$$c^2 = a^2 + b^2 - 2ab\cos C \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad b^2 = a^2 + c^2 - 2ac\cos B$$

To solve for any angle given three sides:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Part B - Concept
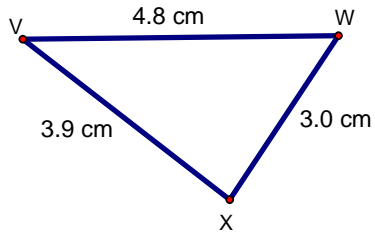
Determine whether you should use the sine law or the cosine law to solve each triangle.

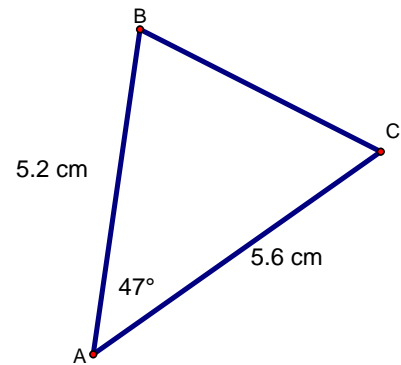| Diagram	Sine Law or Cosine Law?	Reason
a) H — 5.1 cm — I, 3.6 cm, 5.2 cm, G		
b) K, 3.7 cm, 70°, L, 59°, J		
c) N, 5.0 cm, 72°, M, 4.4 cm, O		
d) Q, 5.6 cm, P 45°, 4.8 cm, R		

Part C – Concept Practice

1. Solve for the indicated measure.

a) $\angle W$

4.8 cm

V   W

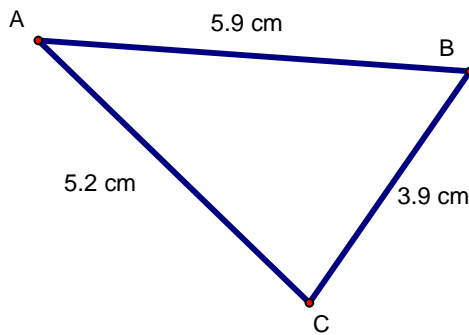3.9 cm   3.0 cm

X

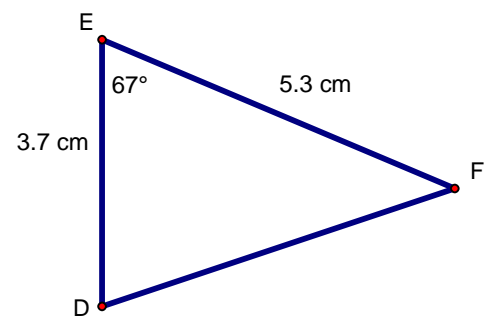b) $\overline{BC}$

B

5.2 cm

C

5.6 cm

47°

A

2. Solve each triangle. HINT: You will need to use the cosine law first. Then, you can use the sine law to solve the rest of the triangle.

a)
A   5.9 cm

B

5.2 cm   3.9 cm

C

b)
E

67°   5.3 cm

3.7 cm

F

D

c)
H

2.5 cm

86°

G   3.6 cm   I

d)
4.6 cm   K

J

3.9 cm

4.9 cm

L

Part D – Applications

1. The distance from Toronto to Sudbury by plane is 390km. The distance from Sudbury to Ottawa is 490 km. If the angle formed at Sudbury (between Toronto and Ottawa) is 53°, what is the distance from Toronto to Ottawa?

2. A hockey player is standing 2.7 m from the left post and 2.4 m from the right post. The net is 1.8 m wide. Within what angle must the hockey player shoot the puck to make it within the goal?
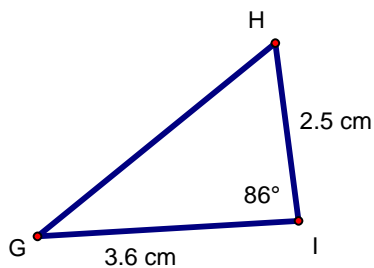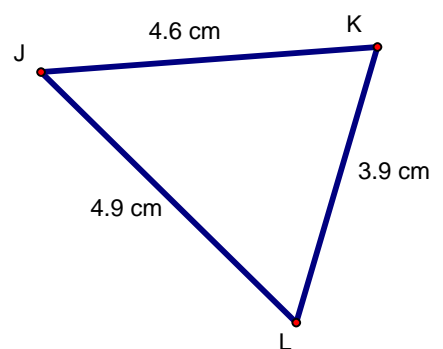
Part E – Applications Practice

1. From your cottage, you can see a bridge that connects from one side of a lake to another. Your cottage is 25.6 m from the left side of the bridge, and 28.7 m from the right side. If the angle from your cottage spans 47° across the bridge, how long is the bridge?

2. The sides of a triangle have a measure of 4.9 m, 5.7 m, and 3.1 m. Determine the measure of the smallest angle. HINT: The smallest angle is across from the smallest side.

3. The distance between the corner pockets of a pool table are 120 cm apart. A player knows that the ball she wants to hit is 80 cm from the left pocket and 100 cm from the right pocket. What angle spans the two pockets from the position of the ball?

4. A sailboat wants to navigate between the shoreline and a hazardous rock. The captain knows that the distance from the boat to the shoreline is 4.5 km and the distance to the rock is 5.7 km. If the angle from the shoreline to the rock is 25°, what is the distance between the shoreline and the rock?